Midterm exam Calculus-3 (10 points free):

Total points to obtain 100

In all problems provide a brief justification for what you do

<u>Problem 1</u> (15 points)

Show that $$\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad \text{with } x \in (-\infty, +\infty)$$

<u>Problem 2</u> (15 points)

Prove that the series $$\sum_{n=1}^{+\infty} n^c \sin\frac{1}{n^c} \quad (c > 1)$$ is divergent!

Tip: see how the a_n term behaves when $n \to \infty$....

<u>Problem 3</u> (20 points)

Consider the series $$\sum_{n=1}^{+\infty} n!(2x-1)^n$$

(a) For which values of x is convergent (10 points)

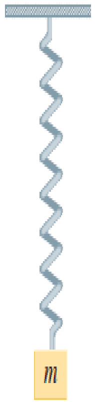
(b) For which values of x is divergent (10 points)

Problem 4 (15 points)

Determine the value of the series: $\sum_{n=0}^{\infty} [x^n + y^n]/w^n \quad (|x|,|y|<|w|)$

Problem 5 (25 points)

Suppose a mass m is attached to a spring with spring constant k, and let $k = m\omega^2$. If an external force $F(t) = F_o \cos(\omega t)$ is applied, then we have the equation of motion in presense of dissipation (m, k, c>0):



$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t) \qquad (1)$$

If we assume $c^2 - 4mk < 0$ show that a particular solution $x_p(t)$ of equation (1) is given by:

$$x_p(t) = \left(\frac{F_o}{c\omega}\right)\sin(\omega t)$$